Idealised GFD Models II

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May 11, 2012

Ocean Modeling

We would like to simulate the ocean to find out how it behaves. To do this we need to

- Write down equations to describe the physics involved
- Solve these equations

Numerically solving a differential equation

Consider the D.E.

$$\frac{dy}{dx} = x^2 \quad (1)$$

$$y(1) = 1 \quad (2)$$

Analytical techniques give

$$y(x) = \frac{x^3 + 2}{3} \quad (3)$$

But what if we didn't have analysis?

Numerical Solution

Definition of the derivative

$$
\begin{aligned}
f'(x) &= \frac{f(x+\Delta x) - f(x)}{\Delta x} && (4) \\
f(x+\Delta x) &= f(x) + f'(x)\Delta x && (5)
\end{aligned}
$$

We can numerically solve our problem using different values of Δx

$$
\begin{aligned}
f'(x) &= x^2 && (6) \\
f(1) &= 1 && (7)
\end{aligned}
$$

$x =$	1	1.25	1.5	1.75	2.0
$\Delta x = 1.00$	1				2
$\Delta x = 0.50$	1		1.5		2.625
$\Delta x = 0.25$	1	1.25	1.641	2.203	2.969
$\Delta x \to 0$	1	1.318	1.792	2.453	3.333

Numerical integration

- Given initial state, $f(t_0)$, and $f'(t)$ we can compute $f(t)$ into the future
- We call this process *numerical integration*
- We can use numerical integration to solve the equations describing physical processes.

QG Equations

Recall the quasigeostropic equations over a horizontal layer.

$$q_t = -(uq)_x - (vq)_y - \frac{f_0}{H}\delta_z(e) + A_2\frac{\nabla_H^4 p}{f_0} \quad (8)$$

$$q = \frac{\nabla_H^2 p}{f_0} + \beta(y - y_0) + \frac{f_0}{H}\delta_z(\eta) \quad (9)$$

$$(u, v) = \frac{1}{f_0}(-p_y, p_x) \quad (10)$$

$$\eta = \frac{(p_- - p_+)}{g'} \quad (11)$$

- If we know q (vorticity), we can compute p (pressure)
- If we know q, p we can compute q_t
- Therefore, if we know q at a particular point in time, we can numerically integrate to find q into the future.

Control variables

As well as the dynamic state variables p, q, the equations have two other types of variables

- Fixed control parameters (f_0, β, H, A_2, g')
- Dynamic coupling variables (e)

By adjusting these variables we can perform different experiments to investigate the nature of the system being modelled.

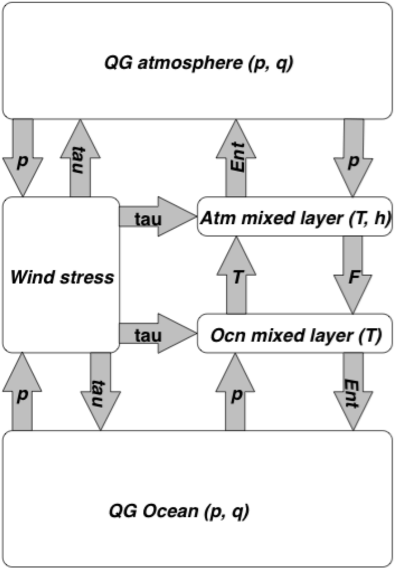
Q-GCM

We will now consider a complete model consisting of 5 components

- QG ocean
- Ocean mixed layer
- Wind stress
- QG Atmosphere
- Atmosphere mixed layer

Each component has state variables, control parameters and dynamic coupling variables.

Component Coupling



Running a Model

- Choose values for control parameters
- Set initial conditions of state variables
- Numerically integrate model equations for a set period of time
- Analyse output

Model Output

Models calculate two types of output:

- Prognostic quantities: The state variables use in the model calculation
- Diagnostic quantities: Values derived from state variables, but not needed for the numerical integration itself

Example Diagnostics

Diagnostics let us investigate physical properties of the system being modelled.

- Average transport
- Average kinetic energy
- Max absolute velocity
- Total convective heat transport
- and many more...

Summary

- Climate models let us numerically integrate equations representing the physical world
- Models are composed of multiple interacting components
- Each component has controls parameters which can be adjusted to investigate different processes
- Diagnostic output lets us obtain useful physical information from the model as it runs